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Low-Energy Kähler Potentials In Supersymmetric Gauge Theories With (Almost) Flat Directions

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Abstract

We derive the supersymmetric low-energy effective theory of the D -flat directions of a supersymmetric gauge theory. The Kähler potential of Affleck, Dine and Seiberg is derived by applying holomorphic constraints which manifestly maintain supersymmetry. We also present a simple procedure for calculating all derivatives of the Kähler potential at points on the flat direction manifold. Together with knowledge of the superpotential, these are sufficient for a complete determination of the spectrum and the interactions of the light degrees of freedom. We illustrate the method on the example of a chiral abelian model, and comment on its application to more complicated calculable models with dynamical supersymmetry breaking.

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1 Introduction

When considering Lagrangians with many different mass scales, it is often useful to integrate out the heavy degrees of freedom and derive an effective low-energy theory of the light fields. In supersymmetric theories, it is useful to follow this procedure while manifestly preserving supersymmetry.

Supersymmetric gauge theories often exhibit directions in the scalar field space, along which the scalar potential identically vanishes, the so-called “flat” directions [1]. When studying the low-energy dynamics of the theory at a given point on the flat direction manifold it is useful to consider an effective Lagrangian where the fields are constrained to these flat directions [2]. However, the vanishing of the D term is a nonholomorphic constraint which generally cannot be solved in terms of chiral superfields. It is therefore not clear a priori how to construct an effective theory of the flat directions which manifestly maintains supersymmetry.

The method for finding the Kähler potential of the low-energy sigma model used by Affleck, Dine and Seiberg (ADS) [2] is based on the procedure of using the flat direction equations to project the full theory Kähler potential without gauge fields onto the light gauge invariant chiral superfields. For simple gauge groups and matter representations, such as $SU(2)$ one-flavor SQCD, it leads to unambiguous results for the Kähler potential. However, even in the case of the simplest model with dynamical supersymmetry breaking, the $SU(3) \times SU(2)$ model [2], the Kähler potential is determined by the solution of a cubic equation [2]. The choice of the correct solution can only be made by examining the positivity of the Kähler metric for each of the roots of this equation [3].

In this paper, we address both of these ambiguities. We derive the ADS effective Lagrangian by applying a *holomorphic* constraint and explicitly integrating out the heavy vector fields. We show how this procedure is equivalent, via a *nonholomorphic* field redefinition, to the ADS procedure.

We also show how to compute all derivatives of the Kähler potential at the flat direction without solving the complicated equations for the vanishing of the D terms. Moreover, our procedure yields a manifestly positive definite Kähler metric, as we show in Sect. 2. It is very general and can be applied to calculable models of dynamical supersymmetry breaking, with virtually unknown ground state properties.

2 Gauge Invariant Description of the Light Degrees of Freedom

In this section we derive the gauge invariant theory of the light degrees of freedom along a flat direction with completely broken gauge symmetry. It can be generalized to the case where there is some unbroken nonabelian gauge group, so long as we are interested in the effective theory at scales below the scale where all particles carrying charge under the unbroken group acquire a dynamical mass. The matter part of the classical Lagrangian of a general supersymmetric gauge theory with gauge group G (of dimension $N - n$) and no classical superpotential has the form of a D-term:

$$L_D = Q^\dagger e^{V^a T^a} Q, \quad (2.1)$$

where Q_i are chiral matter superfields. Hereafter i runs over both the gauge index and the different representations, $i = 1, \dots, N$, where N is the number of chiral matter superfields. V^a , $a = 1, \dots, N - n$, denote the vector superfields, corresponding to the various factors in G . Under a supergauge transformation the matter and gauge superfields transform as [4]:

$$\begin{aligned} Q &\rightarrow e^{-i\Omega^a T^a} Q \\ e^{V^a T^a} &\rightarrow e^{-i\Omega^{a\dagger} T^a} e^{V^b T^b} e^{i\Omega^a T^a}, \end{aligned} \quad (2.2)$$

where the parameters of the transformation Ω^a are chiral superfields. The scalar potential of this theory has classically flat directions along which it identically vanishes. They are given by the solutions of

$$Q^{\dagger i} T_i^a{}^j Q_j = 0, \quad (2.3)$$

where Q_i now means the scalar components of the corresponding chiral superfields and a sum over the different representations is again implicit. If we expand the theory around a solution of (2.3) sufficiently far from the origin the theory is weakly coupled and can be analyzed perturbatively [1]. We consider the theory in the vicinity of a such solution of (2.3), which completely breaks the gauge symmetry [2] (up to possible abelian factors). The number of broken generators of the gauge group is $N - n$. Then $N - n$ of the N chiral superfields Q_i are massive and n chiral superfield are massless (in the absence of superpotential).

Below the scale of the gauge boson masses the heavy gauge fields and their superpartners can be integrated out. As in ref.[2] we assume that the theory of the light degrees of

freedom can be given a gauge invariant description, where the light supermultiplets are represented by a set of gauge invariant chiral superfields X^A , $A = 1, \dots, n$, which are independent polynomials in the matter fields Q_i :

$$X^A = X^A(Q). \quad (2.4)$$

The dynamics of the theory below the scale of the gauge boson masses is described by a supersymmetric Kähler sigma model with coordinates spanned by the light chiral superfields (2.4) [2]. The derivation of the Kähler potential of this sigma model is the focus of this letter.

The derivation is nontrivial, because if one follows the procedure of ref.[2] and applies the nonholomorphic constraint of eq.2.3 it is not clear why the resulting Lagrangian should be supersymmetric. Explicitly, for particular models one observes that the flat directions equations cannot be promoted to chiral superfields. Furthermore, because the constraint equations are real, there is an insufficient number of constraining equations to determine the light chiral fields (see discussion below).

Another difficulty with the ADS procedure is more a matter of practice. To find the Kähler potential of the effective theory, one needs to solve for it along the flat directions, which can be very complicated. Even for the simplest model of dynamical supersymmetry breaking based on $SU(3) \times SU(2)$, one needs to solve a cubic polynomial equation and only the correct root gives a positive definite kinetic energy for the light degrees of freedom.

In the rest of this section, we will show how to derive the ADS potential. We will use holomorphic constraints when separating the light from the heavy degrees of freedom. We will show using our procedure how one can compute derivatives of the Kähler potential (which are all that are required for finding the spectrum and interactions) simply, without explicitly solving for the full form of the low energy Kähler potential.

In order to separate the light and heavy degrees of freedom in a gauge invariant way, it is necessary to make a field redefinition

$$\begin{aligned} Q &\rightarrow e^{-iG^a T^a} Q(X) \\ e^{V^a T^a} &\rightarrow e^{-iG^{c\dagger} T^c} e^{V^a T^a} e^{iG^b T^b}, \end{aligned} \quad (2.5)$$

where G^a are Goldstone chiral superfields, transforming as

$$e^{-iG^a T^a} \rightarrow e^{-i\Omega^a T^a} e^{-iG^b T^b} \quad (2.6)$$

under supersymmetric gauge transformations. The vector superfield \mathcal{V}^a is gauge invariant, as follows from (2.2), (2.5), and (2.6). On-shell it describes a massive vector supermultiplet [4].

We are interested in making the field redefinition (2.5) locally, around a point $Q_i = v_i$ on the flat direction manifold (“moduli space”) where the gauge symmetry is completely broken. Hence the $N - n$ vectors (N -dimensional) $T_i^a{}^j v_j$ are all nonvanishing and linearly independent. We assume that one can find $X^A(Q)$ such that all derivatives $X_i^A(Q) \equiv \partial X^A(Q)/\partial Q_i$ are nonvanishing at this point. The functional independence of $X^A(Q)$ then assures that the n vectors (N -dimensional) $X_i^A(v)$ are linearly independent. Gauge invariance of $X^A(Q)$ implies that $X_j^A(v) T_i^a{}^j v_j = 0$, for any A, a . Taken together with the linear independence, this implies that the N vectors $X_i^{A*}(v^*) \equiv \partial X^{A*}(Q^*)/\partial Q^{*i}|_{Q^*=v^*}$ and $T_i^a{}^j v_j$ form a complete basis in the complex space spanned by Q_i . The gauge invariant fields X^A have the expansion around $Q_i = v_i$

$$X^A = X^A(v_i) + x^A . \quad (2.7)$$

Then the expansion of $Q(X)$ around points on the flat direction manifold is

$$Q_i(X) = v_i + q_i(x) . \quad (2.8)$$

The most general form of $q_i(x)$ is

$$q_i(x) = X_i^{A*}(v) \lambda_A(x) + T_i^a{}^j v_j \chi_a(x) . \quad (2.9)$$

An important application of this formalism is to theories which possess flat directions only in a certain limit, the so-called almost flat directions. Examples of such models are massive supersymmetric QCD (with masses much smaller than the strong coupling scale of the theory) [1], the $SU(2) \times SU(3)$ [2], and the calculable $SU(5)$ [5] models of dynamical supersymmetry breaking. For our purposes, the common property of these models is that the superpotential can be considered as a perturbation so long as the scale of the vacuum expectation values along the flat directions is larger than the strong coupling scale of the theory. The superpotential is a gauge invariant holomorphic function [6] of the chiral superfields Q_i , $W = W(X(Q))$. After the field redefinition (2.5), by gauge invariance of W , the resulting superpotential is independent of the Goldstone superfields:

$$W = W(X(Q(X))) = W(X) . \quad (2.10)$$

Here we required that $Q(X)$ obey

$$X(Q(X)) = X . \quad (2.11)$$

Notice that requiring (2.11) allows a nonholomorphic $\mathcal{Q}(X^\dagger, X)$ of a special form

$$\mathcal{Q}(X^\dagger, X) = e^{\mathcal{B}^a(X^\dagger, X)T^a} Q(X) , \quad (2.12)$$

with $\mathcal{B}^a(X^\dagger, X)$ an arbitrary complex function. Since $X(Q)$ is invariant under the complex extension of the gauge group, the nonholomorphic factor disappears from (2.11).

With the most general form of $Q_i(X)$ given in terms of the N functions $q_i(x)$ (2.9), the n holomorphic functions $X(Q)$ can only be inverted after imposing $N - n$ holomorphic constraints. The field redefinition (2.5) amounts to introducing new coordinates on the space of the chiral superfields spanned by Q_i :

$$\{Q_i\} \rightarrow \{X^A(Q), S^a(Q)\} , \quad (2.13)$$

the only condition on $S^a(Q)$ being the nondegeneracy of the mapping (2.13):

$$\det \left(\frac{\partial X^A(Q)}{\partial Q_i}, \frac{\partial S^a(Q)}{\partial Q_i} \right) \neq 0 . \quad (2.14)$$

In the vicinity of $Q_i = v_i$ the additional $N - n$ coordinates S^a are

$$S^a = v^{*j} T_j^{a \ i} Q_i , \quad (2.15)$$

since $\det(X_i^A(v), v^{*j} T_j^{a \ i}) \neq 0$. Now the holomorphic functions $X(Q)$, obeying (2.11) can be inverted by imposing $N - n$ holomorphic constraints. We choose the constraints

$$S^a = v^{*j} T_j^{a \ i} Q_i = 0 , \quad (2.16)$$

since then the fields $Q(X)$ (2.17) are flat to linear order, $Q^* T^a Q = 0 + O(x^2)$ (this follows from gauge invariance of $X^A(Q)$).

Notice that this equation can be interpreted as the fact that the projection of the light field on the Goldstone boson direction vanishes. Then the fields Q_i (2.8), (2.9) obeying the constraint (2.16) have the expansion

$$Q_i(X) = v_i + X_i^{A*} \lambda_A(x) . \quad (2.17)$$

The n functions $\lambda_A(x)$ are determined by inverting (2.11)

$$x^A = X^A(v_i + X_i^{B*} \lambda_B(x)) - X^A(v_i) , \quad (2.18)$$

in terms of a Taylor series in x^A :

$$\lambda_A(x) = (X_i^C X_i^{*D})_{AB}^{-1} x^B + O(x^2). \quad (2.19)$$

Expressing the Lagrangian (2.1) in terms of the new fields $Q(X)$ and \mathcal{V} , and expanding in powers of the heavy field \mathcal{V} , we obtain

$$\begin{aligned} L_D &= Q(X)^\dagger e^{\mathcal{V}^a T^a} Q(X) \\ &= Q(X)^\dagger Q(X) + \mathcal{V}^a Q(X)^\dagger T^a Q(X) + \dots , \end{aligned} \quad (2.20)$$

where dots denote higher powers of the massive vector superfield¹. Below the scale of the mass of the gauge fields, the heavy vector multiplet can be integrated out. Since we are only interested in the leading term of the low-energy expansion, we can neglect the kinetic term of the gauge field. The zero-momentum tree graphs are easily computed by perturbatively solving the equation of motion for \mathcal{V} that follow from (2.20). Then the low-energy Kähler potential $K(X^\dagger, X)$ is given by (2.20) with the field \mathcal{V}_c substituted by the solution to its classical equation of motion, i.e.

$$K = Q^\dagger(X) e^{\mathcal{V}} Q(X)|_{\mathcal{V}=\mathcal{V}_c} , \quad (2.21)$$

with \mathcal{V}_c determined by

$$\left. \frac{dK}{d\mathcal{V}} \right|_{\mathcal{V}=\mathcal{V}_c} = 0 . \quad (2.22)$$

If the functions $Q(X)$ obey the flat direction equations (2.3) there are no additional tree-level contributions to the low-energy Kähler potential, as follows from (2.20)². In general, however, the holomorphic functions (2.17), (2.18) do not obey the flat directions equations. This is the case, e.g. in the $SU(3) \times SU(2)$ model with dynamical supersymmetry breaking

¹The gauge invariant field \mathcal{V} , defined by (2.5), includes the Goldstone fields and their superpartners and therefore there are infinitely many terms in the expansion of $e^{\mathcal{V}}$.

²In the one-flavor $SU(2)$ SQCD finding such a redefinition is simple. Let Q and \bar{Q} be the two doublets of chiral matter transforming in the 2 and 2* representations. For dimensional reasons $Q(X) = \sqrt{X} \eta$ and $\bar{Q}(X) = \sqrt{X} \xi$, with η and ξ being two constant spinors with unit norm, obeying $\eta^\dagger T^a \eta - \xi T^a \xi^\dagger = 0$. Then the Kähler potential $K(X^\dagger, X)$ becomes $2\sqrt{X^\dagger X}$, which coincides with the potential derived in [2].

[3] and in the simple abelian chiral model considered in the next section. Then there are zero-momentum tree-level graphs due to the heavy vector supermultiplet, which contribute to the low-energy Kähler potential.

Since the holomorphic functions $Q(X)$ are flat up to quadratic order in the expansion around the flat direction, the leading contribution of the tree graphs is fourth order in x^A . So when computing derivatives of fourth order or higher one needs to incorporate the contribution to the potential from the nonzero vector field. However, to compute the second and third derivatives, it is sufficient to invert Q in terms of X . The Kähler metric at the minimum does not receive tree-level corrections from the heavy fields and is given by the manifestly positive definite expression

$$K_{A^*B} = (X_i^C X_i^{*D})^{-1}_{AB} . \quad (2.23)$$

Similarly, using (2.17), (2.18) and (2.21), one can derive expressions involving three derivatives. However, because the Q fields we define are not D -flat, to calculate fourth or higher order derivatives of the Kähler potential at the minimum, one needs to incorporate the additional contribution from integrating out the vector field.

However, there does exist a nonholomorphic redefinition of the fields such that $\tilde{Q}_i(X^\dagger, X)$ obey the (nonholomorphic) flat direction equations (2.3). The Kähler potential (2.21) can be written as

$$K = Q^\dagger(X) e^{\mathcal{V}_c/2} e^{\mathcal{V}'} e^{\mathcal{V}_c/2} Q(X) \Big|_{\mathcal{V}'=0} \quad (2.24)$$

Defining the new fields

$$\tilde{Q}(X^\dagger, X) \equiv e^{\mathcal{V}_c(X^\dagger, X)/2} Q(X) , \quad (2.25)$$

the Kähler potential (2.24) becomes

$$\tilde{K} = \tilde{Q}^\dagger(X^\dagger, X) e^{\mathcal{V}'} \tilde{Q}(X^\dagger, X) \Big|_{\mathcal{V}'=0} . \quad (2.26)$$

Now, from (2.26) it follows that

$$\frac{d\tilde{K}}{d\mathcal{V}'^a} \Big|_{\mathcal{V}'=0} = \tilde{Q}^\dagger(X^\dagger, X) T^a \tilde{Q}(X^\dagger, X) = \frac{dK}{d\mathcal{V}} \Big|_{\mathcal{V}=\mathcal{V}_c} = 0 , \quad (2.27)$$

where the second equality follows from the redefinition (2.25) and the last holds by virtue of (2.22). Equation (2.27) shows that the nonholomorphic functions (2.25) $\tilde{Q}(X^\dagger, X)$ obey the

flat direction equations. These fields correspond to those of Affleck, Dine, and Seiberg, [1], and differ from the fields Q (2.17) at quadratic order in x^A .

Note that the D -flat conditions only give $N - n$ real constraints whereas we need $N - n$ complex ones, hence there is an insufficient number of constraints. In the ADS construction the $N - n$ additional real constraints correspond to fixing the gauge.

The important fact is that even though the fields \tilde{Q} are not holomorphic, the superpotential constructed from these fields is nonetheless manifestly supersymmetric. This is because the low-energy superpotential is still a holomorphic function of the light superfields, since by gauge invariance, the nonholomorphic factor $e^{\mathcal{V}_c(X^\dagger, X)/2}$ in the redefinition drops out of the superpotential. This justifies the ADS construction.

Therefore, when deriving the effective theory, one is faced with several possibilities. One can apply a holomorphic constraint to restrict oneself to the light degrees of freedom. It is easy with this procedure to derive all derivatives of the Kähler potential³. When applying this procedure to the derivatives of fourth or higher order, one needs to include the contribution from explicitly integrating out the heavy vector fields by solving their equations of motion.

Alternatively, one can use nonholomorphic fields which exactly satisfy the D flat equations. This procedure is justified by the fact that the nonholomorphic field contribution will cancel out from gauge invariant superpotential terms. Here, the classical vector field is zero, so there are no additional contributions to the Kähler potential for the light fields. The leading order derivative terms are the same as those calculated with the previous procedure. The higher order terms can be computed perturbatively as well; these are the same as those derived from the Lagrangian with a holomorphic constraint, so long as the classical vector field contribution is incorporated.

We should also note that if there is some unbroken nonabelian gauge group, our procedure can be applied, as long as we are interested in the effective theory below the dynamically generated mass scale of the fields carrying gauge charge with respect to the unbroken gauge group. So long as we can find $X^A(Q)$ such that $T_j^\alpha{}^i X_i^A(v) = 0$ holds for the unbroken generators T^α , it is easy to see that the fields (2.17) obey the unbroken group D -flat equations and are decoupled from the (strongly interacting) gauge field of the unbroken group.

³Alternatively, one can explicitly solve the holomorphic constraint equation, similar to the procedure of ADS (as in the one-flavor SU(2) example above).

3 The Low-Energy Kähler Potential: an Abelian Example

In this section we illustrate our method on a simple abelian chiral model. The advantage is that we can solve the vector field equations of motion to all orders and explicitly demonstrate the equivalence of our procedure with that of ADS via a nonholomorphic field redefinition.

Consider a chiral U(1) supersymmetric gauge theory with three chiral matter superfields: S_1 , S_2 of charge 1, and T , of charge -2. The Lagrangian of the model is

$$L_D = S_1^\dagger e^V S_1 + S_2^\dagger e^V S_2 + T^\dagger e^{-2V} T. \quad (3.1)$$

Along the flat directions, given by

$$S_1^\dagger S_1 + S_2^\dagger S_2 - 2T^\dagger T = 0, \quad (3.2)$$

the gauge symmetry is broken. There are two independent gauge invariant chiral superfields in this theory

$$X_1 = S_1^2 T, \quad X_2 = S_2^2 T. \quad (3.3)$$

Performing the field redefinition (2.5), with $Q_i(X)$ determined by (2.17), (2.18), the low-energy Kähler potential is given by (3.1), where the corresponding functions $S_1 = S_1(X_1, X_2)$, etc., are substituted, and V denotes the heavy vector multiplet. Substituting the solution to the equation of motion for V ,

$$e^V = \left(\frac{2T^\dagger T}{S_1^\dagger S_1 + S_2^\dagger S_2} \right)^{1/3}, \quad (3.4)$$

into (3.1), the low-energy Kähler potential becomes

$$K(X^\dagger, X) = 3 \left(\frac{\sqrt{(S_1^2 T)^\dagger S_1^2 T} + \sqrt{(S_2^2 T)^\dagger S_2^2 T}}{2} \right)^{2/3}. \quad (3.5)$$

Finally, recalling that $S_1(X_1, X_2)$, etc., obey (2.11), e.g. $S_1^2(X_1, X_2) T(X_1, X_2) = X_1$, we find the expression for the Kähler potential of the light degrees of freedom

$$K(X^\dagger, X) = 3 \left(\frac{\sqrt{X_1^\dagger X_1} + \sqrt{X_2^\dagger X_2}}{2} \right)^{2/3}. \quad (3.6)$$

This coincides with the Kähler potential obtained by the method of [2].

Notice that the vector field contribution only affected fourth and higher order derivatives, since its expansion in light fields begins at second order, as follows from the fact that the fields (2.17) are D flat to linear order. In more complicated examples, where one cannot explicitly solve the vector field equation of motion to all orders, one can nonetheless perturbatively derive the vector field contribution. Explicitly, the scalar contribution corresponds to integrating out the auxiliary complex scalar component (which vanishes in the Wess-Zumino gauge; it is denoted by $M + iN$ in ref.[4]) of the vector field.

4 Conclusion

In this letter we developed a procedure for finding the Kähler potential of the light degrees of freedom in supersymmetric theories, where the gauge symmetry is completely broken along a flat direction of the D -term scalar potential. The resulting Kähler potential is determined as a power series expansion around the given point of the flat direction. The Kähler metric is manifestly positive definite.

The method is quite general and can be applied to any calculable model of dynamical supersymmetry breaking. It is satisfying that one can derive the low energy theory without exactly solving the flat directions equations in terms of the gauge invariant superfields, particularly in the case of more complicated models. This might prove useful when deriving the physics of specific models. Of particular interest is the $SU(5)$ model with two generations [5], since little is known about its ground state or broken symmetries.

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